The Nasdaq-100® Volatility Index: VOLQ
A Methodology Reference Guide for Retail and Institutional Investors

Introduction

Volatility is a measure of the magnitude of price movement—up or down—of a financial instrument like an index or an equity. Historical volatility is a measure of the standard deviation of price changes over a past, fixed time period. In contrast, implied volatility estimates the expected future movement of prices over a specified time horizon of one month, or even one day.

This reference guide answers the questions: What is implied volatility and how is it calculated?

Volatility Products

Volatility products (cash-settled options and futures) have become more prominent over the past decade. They provide investors with tools to manage portfolio volatility risk. A common way to observe volatility is with an index, which tracks the implied volatility of an underlying equity index by using exchange-listed options prices.

Index options are a type of financial product that give the holder the right to buy or sell the price level of the underlying index at a predetermined price (strike price), within a specific time interval (time to expiration). The price of an option is calculated as a function of the current index price, strike price, risk-free interest rate, time to expiration, and volatility; these collective values are used as inputs to various options pricing models. Since index option prices are publicly available in the open market, they can be used to solve for the future index volatility implied by those option prices.

Forward looking, expected volatility is a central driver of index options prices—the higher the expected volatility, the higher the index option price, all else being equal. Using this approach, the market's assessment of future volatility rests on observation of index options prices.

Introducing the Nasdaq-100 Volatility Index: VOLQ

The Nasdaq-100 Volatility Index ("VOLQ") measures 30-day implied volatility of the Nasdaq-100 Index (ticker symbol NDX).

VOLQ is expressed as an annualized percentage and is positively correlated to NDX options prices (both calls and puts). The resulting value broadcasts the expected NDX index trading range over the next 30 consecutive days.

The VOLQ methodology is provided by Nations Indexes, Inc. in partnership with Nasdaq.

For example, if VOLQ is at a price level of 17.90, dividing by the square root of 12 (reflecting the number of 30-day periods in one year) implies an NDX trading range unlikely (with 68% certainty) to rise or fall more than 5.17% over the next 30-day period. Assuming NDX is at a price level of 9000, VOLQ indicates that the aggregate marketplace view is that NDX will have a potential trading range over the next 30 days contained within a range of up 5.17% (9465) to down 5.17% (8535).

Generally, more uncertainty in the outlook for NDX tends to cause options prices to rise. This is because there is a greater probability that the price will move above or below the strike price. In addition, options are used as insurance to protect against large movements in NDX price; investors buy options to hedge their portfolio positions against these large price movements (volatility movements), causing options prices to increase. As such, when NDX options prices are higher, VOLQ will be higher, and vice-versa.

Uncertainty in the outlook for NDX → **Options prices increases** → **VOLQ increases**

Nasdaq -100 vs VOLQ

NASDAQ -100   VOLQ
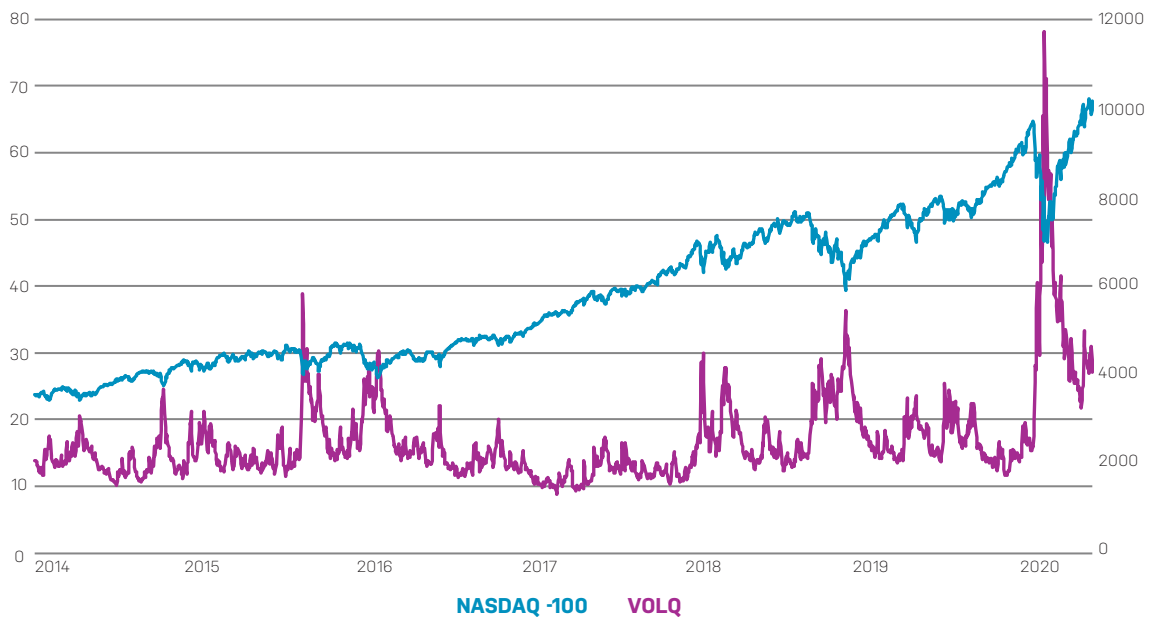
In the chart above, for the period 2014-2019, VOLQ movement is negatively correlated to NDX (-81.49%). VOLQ tends to spike during market downturns and generally declines steadily during bull market moves.

Methodology

VOLQ is calculated throughout the trading day using published, real-time bid and ask quotes on 32 of the most liquid NDX options. VOLQ is based on options with a strike price exactly equal to the forward price of the underlying instrument. These options are considered precisely at-the-money, what traders look at most.

VOLQ Step-by-Step Calculation

The calculation of VOLQ is a function of at-the-money (ATM) NDX options prices, the forward price, the risk-free rate, and the time to expiration. The following steps explain these concepts and illustrate how to calculate the value of the Nasdaq-100 Volatility Index.

STEP 1: CALCULATE THE TIME TO EXPIRATION FOR FOUR CONSECUTIVE WEEKLY EXPIRING OPTIONS.

Time to expiration is critical in the pricing of any option. For American-style exercise options, the holder of the option has the right to exercise the option at any time before this date. If they do this, it likely means they are speculating on or hedging against price fluctuation before the option expires. In contrast, VOLQ uses only European-style exercise options, which confer the right to exercise the option expressly on the exact expiration date.

VOLQ measures 30-day implied volatility—the expected volatility over the next 30 days. However, options expiring exactly 30 days from the current date with a strike price exactly equal to the index price ("at-the-money") are not always available; prices are continually fluctuating throughout the day. As such, other listed options are used in the calculation of a 30-day at-the-money option price. VOLQ evaluates all NDX options throughout the trading day to select certain listed options; these options are then used to calculate the synthetic price of a 30-day precisely at-the-money option.

There are eight selected options—four calls and four puts—for each of four consecutive weekly expirations used in this calculation. Two expiries are less than 30 days from the current date and two expiries are greater than 30 days from the current date. Accordingly, the constituent options expire in the following manner:

OPTION	TIME TO EXPIRATION DAY (FROM CURRENT DATE)
First-Term Expiration	16-22 days
Second-Term Expiration	23-29 days
Third-Term Expiration	30-36 days
Fourth-Term Expiration	37-43 days

The number of days from the current date to the date of the first-term expiration is a range because options expire on Fridays. This means that the amount of time until the first-term expiration varies based on the current day of the week.

For example, to calculate VOLQ at 11:28 A.M. ET on Monday, July 30, 2018, the first-term expiration would be the close of trading on Friday, August 17. This is the standard monthly expiration, which occurs at 9:30 A.M. on the third Friday of the month. The second- (seven days from August 17), third- (14 days), and fourth- (21 days) term expirations would be the next consecutive weekly expiries, respectively. For those weekly expirations not expiring on the third Friday of the month, options are set to expire at 4:00 P.M. ET.

1. Friday, August 17, 2018 (first term)
2. Friday, August 24, 2018 (second term)
   August 29, 2018 – 30 days from July 30, 2018
3. Friday, August 31, 2018 (third term)
4. Friday, September 7, 2018 (fourth term)

Specifically, the calculation of implied volatility uses the time to expiration expressed as a fraction of the number of minutes in a year. The first step to solving for the value of VOLQ is to calculate the fractions (T1 below) representing the time to each of the four expirations.

To calculate VOLQ on Monday, July 30, 2018, at 11:28A.M. ET whereby the first term expiration is Friday, August 17, 2018, the number of minutes (M_1 below) to expiration of the first term option would be:

$$M_1 = 752\ (minutes\ remaining\ on\ July\ 30) + 24{,}480\ (17\ full\ days\ remaining * 1440\ minutes\ per\ day)$$
$$+ 570\ (minutes\ from\ midnight\ to\ 9{:}30\ am\ ET\ on\ August\ 17) = 25{,}802$$

Represented as a fraction of the number of minutes in a year, we find T_1:

$$T_1 = \frac{25{,}802}{525{,}600} = 0.0490906$$

This calculation is repeated for each of the four expirations.

STEP 2: CALCULATE THE FORWARD PRICE FOR EACH TERM.

The forward price represents the expected future price of an asset based on put/call parity. It is calculated for each of the four expiries used in step one.

The relevant NDX option prices used in VOLQ construction are determined by the NBBO (National Best Bid and Offer) mid-point between the best bid (highest bid price) and best ask (lowest ask price) in the NDX options market. NDX options are listed on the options exchanges: Nasdaq PHLX LLC ("PHLX"), Nasdaq ISE LLC ("ISE"), and Nasdaq GEMX LLC ("GEMX"). Only strike prices divisible by 25 are used in the calculation of VOLQ because these strikes are more consistently available and attract the most investors.

The first step to calculating the forward price is to identify the strike price for which the absolute difference between the price of the call and the price of the put is smallest. This strike price and the price of the associated options are used in the equation:

$$F = K^* + e^{R*T} * (call(K^*) - put(K^*))$$

F is the forward price for the given term. K* is the strike price which displays the smallest absolute difference between the price of the call option and the price of the put option. R is the risk-free rate. The risk-free instrument is the U.S. Treasury bill with a maturity date closest to the expiration date of the options. T is the time to expiration (calculated in step 1). Call(K*) is the price of the call option for strike price K and put(K*) is the price of the put option for strike price K.

Continuing with the example in step one, for the first-term expiration, the strike price with the smallest absolute difference between put price and call price is the 7200 strike (see August 17 expiration table below). Applying a risk-free rate of

$$F_1 = 7200 + e^{(0.01950*0.0490906)} * (122.250 - 114.350) = 7207.9076$$

1.950%, the forward price calculation for the first term expiry is:

Where 122.25 is the price of the call option with a 7200 strike and 114.35 is the price of the put option with a 7200 strike.

STEP 3. DETERMINE THE PRECISELY AT-THE-MONEY CALL AND PUT OPTION PRICES.

To get to the precisely at-the-money option price, a total of 32 options are used—16 calls and 16 puts. For each term, four different strike prices are used for both a call and a put as shown in the diagram below:
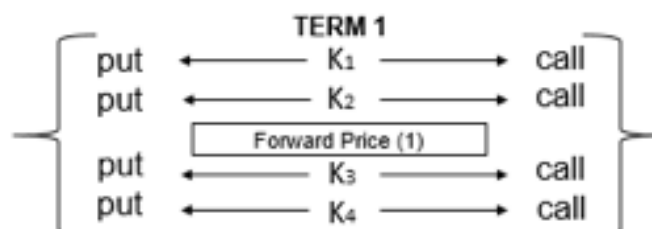
The forward price (step two) informs which options are used to get to the synthetic precisely at-the-money option price. The two strike prices immediately above the forward price (K_1 and K_2) and the two strike prices immediately below the forward price (K_3 and K_4) are used. This process is repeated for each term.

Given the forward price calculated above (7207.9076) and the options listed below for the August 17, 2018, expiration, four strike prices are selected.

AUGUST 17 EXPIRATION

STRIKE PRICE	CALL BID	CALL ASK	CALL MIDPOINT	PUT BID	PUT ASK	PUT MIDPOINT	DIFFERENCE
7150	151.30	155.70	153.50	94.30	97.40	95.85	57.65
7175	135.40	139.50	137.45	103.30	106.30	104.80	32.65
7200	120.40	124.10	122.25	112.60	116.10	114.35	7.90
7225	105.90	109.40	107.65	123.10	126.70	124.90	17.25
7250	92.40	95.70	94.05	134.20	138.20	136.20	42.15
7275	79.70	82.70	81.20	146.30	150.70	148.50	67.30

As such, the 7175, 7200, 7225, and 7250 call and put options are applied to this example. The precisely at-the-money option price is a weighted average of the option prices at these strikes. This means that each strike is multiplied by a factor reflecting its importance. Calculating this factor requires two steps: 1. Determine the raw weightings; and, 2. Normalize the weightings.

This diagram illustrates how the different strikes are weighted in terms of their importance when calculating the precisely at-the-money option price.
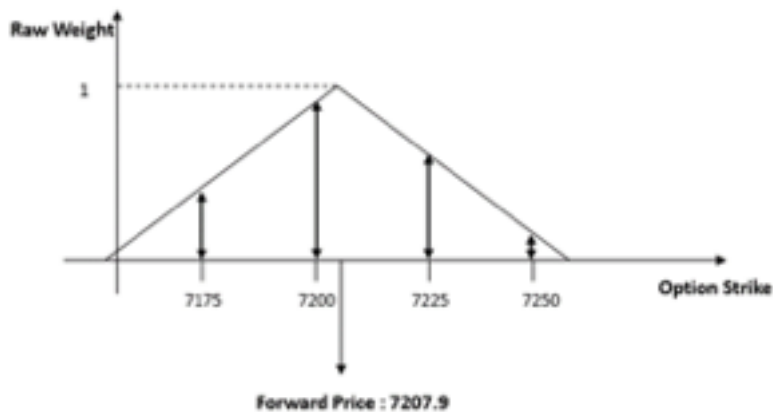
Calculate the raw weightings by following this progression:

If:

$$\left| \frac{K_n - F}{50} \right| \leq 1$$

Then:

$$W_n = 1 - \left| \frac{K_n - F}{50} \right|$$

Else:

$$W_n = 0$$

Where K_n is the strike price and n indicates which of the four strikes (7175, 7200, 7225, or 7250) is being weighted; F is the forward price for the given expiry; W is the raw weighting for strike price n.

The number 50 is used as a scaling factor for the strikes. Fifty was chosen because it is the maximum difference between any strike and the forward price (the numerator). The lowest strike price will be no more than 50 points below the forward price, and highest strike price will be no more than 50 points above the forward price.

Applying the 7200 strike for the first term to this formula gives:

$$\left| \frac{7200 - 7207.9076}{50} \right| \leq 1$$

Therefore:

$$W_1 = 1 - \left| \frac{7200 - 7207.9076}{50} \right| = 0.842$$

Where 7200 is the first out-of-the-money call price (or first in-the-money put price); 7207.9076 is the forward price for the first-term expiry; 50 is the smoothing bandwidth; W_1 is the raw weighting for the 7200 strike.

After calculating the raw weightings for the four strike prices in each term, the weightings are normalized. This step is necessary because the raw weights do not sum to one, meaning they cannot be used to calculate the weighted average option price yet.

The normalized weighting for the 7200 strike in the first term is:

$$w_2 = \frac{W_2}{W_1 + W_2 + W_3 + W_4} = 0.421$$

Where w_1 is the normalized weighting of the first out-of-the-money put price (or first in-the-money call price); W_1 is the corresponding raw weighting; W_2 is the raw weighting for the second out-of-the-money put price; W_3 is the first in-the-money put price; W_4 is the second in-the-money put price.

After each strike price is weighted and normalized, the precisely ATM call option and put option prices can be calculated. Referencing the table above, the prices of the constituent options are used in the formula:

$$ATM\ Call\ Option\ Price_1 = (call(K_1) * w_1) + (call(K_2) * w_2) + (call(K_3) * w_3) + (call(K_4) * w_4)$$

$$ATM\ Put\ Option\ Price_1 = (put(K_1) * w_1) + (put(K_2) * w_2) + (put(K_3) * w_3) + (put(K_4) * w_4)$$

Applying this equation to the example means the sum of each weighted price is used to create the at-the-money price. The price of each option is determined by the midpoint between the best bid and best ask found in the table above. As such, the weighted prices are:

$$(call(K_1) * w_1) = (price\ of\ the\ call\ with\ 7175\ strike * normalized\ weight\ of\ 7175\ strike) = 137.45 * 0.1709243$$

$$(call(K_2) * w_2) = (price\ of\ the\ call\ with\ 7200\ strike * normalized\ weight\ of\ 7200\ strike) = 122.25 * 0.4209243$$

$$(call(K_3) * w_3) = (price\ of\ the\ call\ with\ 7225\ strike * normalized\ weight\ of\ 7225\ strike) = 107.65 * 0.3290757$$

$$(call(K_4) * w_4) = (price\ of\ the\ call\ with\ 7250\ strike * normalized\ weight\ of\ 7250\ strike) = 94.05 * 0.0790757$$

The sum of the weighted prices gives the at-the-money call option price for the first expiration:

$$ATM\ Call\ Option\ Price_1 = (137.45 * 0.1709243) + (122.25 * 0.4209243) + (107.65 * 0.3290757) + (94.05 * 0.0790757) = 117.8136$$

STEP 4: CALCULATE CLOSED-FORM IMPLIED VOLATILITY FOR EACH ATM OPTION.

Using a closed-form equation to calculate implied volatility generates an exact result. This is a precise measurement that differs from traditional measures of implied volatility, which are calculated through iterative trial and error methods.

For example, the diagram below illustrates how eight options from each term are used to arrive at two values for closed-form implied volatility (CFIV)–one for put options and one for call options.
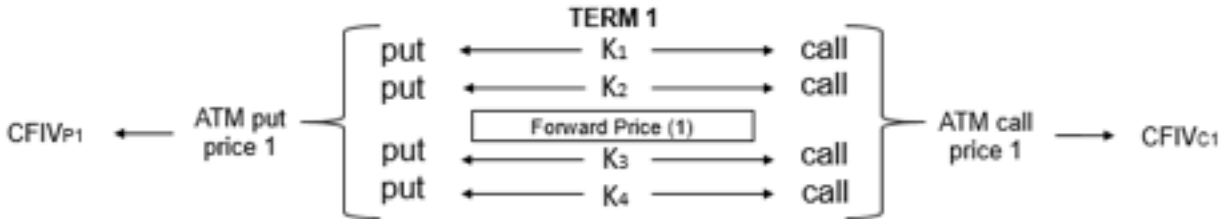
For a given option (call or put) and term, closed-form implied volatility is calculated by simply using the equation:

$$CFIV = \frac{\sqrt{2\pi}}{\left(\frac{F}{e^{R*T}} * \sqrt{T}\right)} * Precisely\ ATM\ Option\ Price$$

The aforementioned formula is a minor variant of the Brenner Subrahmanyan formula to calculate implied volatility. [1]

Applying this equation will result in eight CFIV values.

For the first-term call option (assuming a risk-free rate of 1.950%), CFIV is:

$$CFIV_{C1} = \frac{\sqrt{2\pi}}{\left(\frac{7207.9076}{e^{0.0195*0.0490906}} * \sqrt{0.0490906}\right)} * 117.8136 = 0.185094$$

STEP 5: CALCULATE THE EXPIRATION TOTAL VARIANCE.

Variance is a statistical measure of how much a set of observations differ from the mean. As such, the variance of the constituent call and put options measures the magnitude of price changes (how volatile it is) on a regular basis. The total variance in price for each option is calculated based on the time to expiration (step one) and the closed-form implied volatility (step four). The formula for total variance is:

$$TV = T * CFIV^2$$

Eight different total variances are calculated; one for each precisely at-the-money call and precisely at-the-money put at each of the four expirations. The total variance for the first-term call option is:

$$TV_{C1} = 0.0490906 * 0.185094^2 = 0.00168184$$

Next, the expiration total variance is calculated using the simple average of the call total variance and the put total variance for that expiration. This will result in four values for total variance, one for each of the four expirations. For the first-term, total variance is:

$$TV_1 = \frac{TV_{C1} + TV_{P1}}{2}$$

$$TV_1 = \frac{0.00168184 + 0.00168480}{2} = 0.00168332$$

This is done for each expiration. The four total variance values, one for each expiration, are then interpolated to arrive at a 30-day total variance value. The diagram below depicts how the initial option prices lead to the calculation of four total variances.
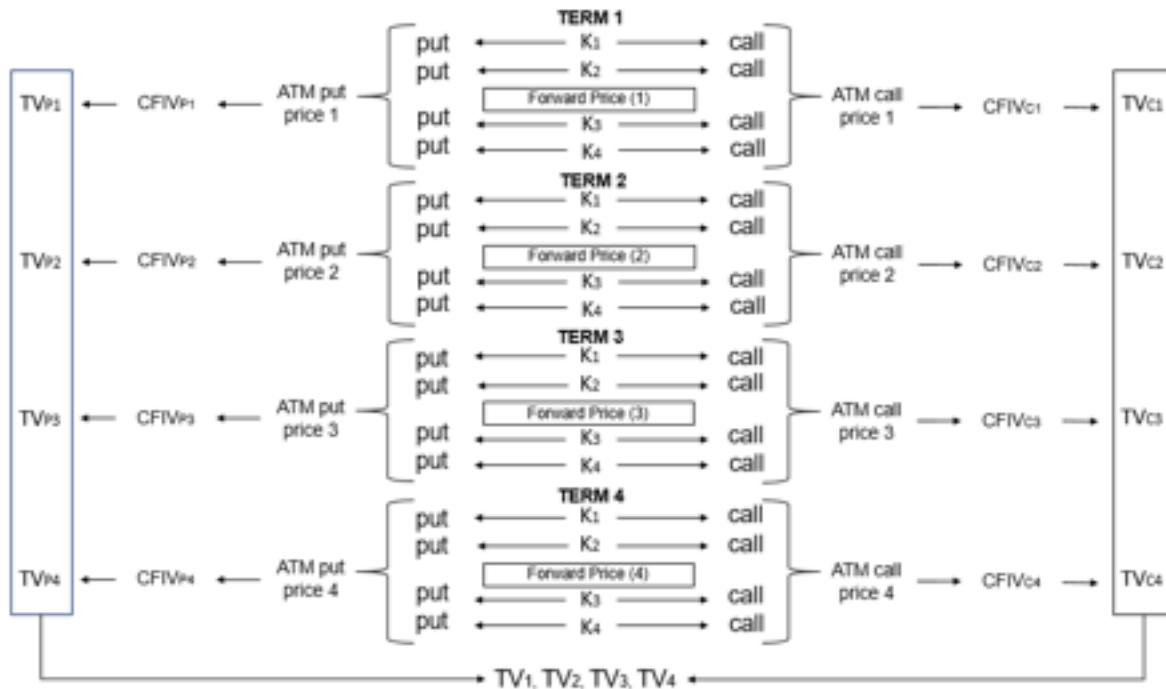
STEP 6: CALCULATE 30-DAY TOTAL VARIANCE.

VOLQ is a 30-day implied volatility index; as such, 30-day total variance is an integral input. The next step is to use the expiration total variances to derive the 30-day total variance. Ultimately, this is a weighted average.

Similar to the calculation of strike price weights, each of the expirations is weighted. The weightings are illustrated in the diagram below:
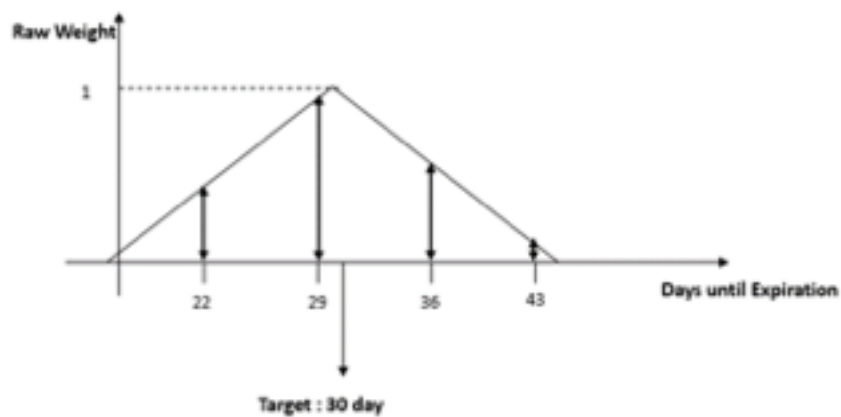
There are 43,200 minutes in 30 days. The scaling factor for the expirations is 15 days (21,600 minutes), expressed as a fraction of a year (525,600 minutes). First, the raw weightings are calculated:

If:

$$\left| \frac{T_1 - (\frac{43,200}{525,600})}{(\frac{21,600}{525,600})} \right| \leq 1$$

Then:

$$W_1 = 1 - \left| \frac{T_1 - (\frac{43,200}{525,600})}{(\frac{21,600}{525,600})} \right|$$

Else:

$$W_1 = 0$$

Where T1 is the time to the first term expiration, expressed as a fraction of a year in minutes, W1 is the raw weighting for the first-term expiration.

Therefore, the raw weighting for the first-term expiry is:

$$W_1 = 1 - |-0.8054630| = 0.1945370$$

The normalized weights are then calculated using the same formula used for the normalized weights of the strike prices. The first-term normalized weight is:

$$w_1 = \frac{0.1945370}{0.1945370 + 0.6792593 + 0.8540741 + 0.3874074} = 0.0919676$$

Where w_1 is the normalized weight for the first term expiry; 0.1945370 is the raw weighting for the first-term expiry; 0.679 is the raw weighting for the second term expiry; 0.854 is the raw weighting for the third term expiry; 0.387 is the raw weighting for the fourth term expiry.

Now there are four total variances, one for each consecutive weekly expiry. The precise 30-day total variance is a weighted average of the total variance, calculated with the normalized weightings. Each expiration total variance is multiplied by its respective weighting and summed to get the 30-day total variance:

$$TV_{30-day} = (TV_1 * w_1) + (TV_2 * w_2) + (TV_3 * w_3) + (TV_4 * w_4)$$

$$TV_{30-Day} = (0.00168332 * 0.0919676) + (0.00228178 * 0.3211206) + (0.00284554 * 0.4037645) + (0.00334221 * 0.1831473) = 0.00264858$$

STEP 7: CALCULATE 30-DAY CLOSED-FORM IMPLIED VOLATILITY.
As depicted in step five, the 30-day closed-form implied volatility is based on the 30-day total variance. The formula is:

$$CFIV_{30-day} = \sqrt{\frac{TV_{30-day}}{(\frac{43200}{525600})}}$$

Applying the example gives the value for 30-day closed-form implied volatility on Monday, July 30, 2018, at 11:28A.M. ET.

$$CFIV_{30-Day} = \sqrt{\frac{0.00264858}{\left(\frac{43,200}{525,600}\right)}} = 0.1795116$$

STEP 8: CALCULATE VOLQ.

The final step in the calculation of VOLQ is to convert 30-day closed-form implied volatility to an annualized percentage (multiply by 100). This index value represents the expected annualized price action in percentage terms (up or down) over the next 30-days.

$$VOLQ = 0.1795116 * 100 = 17.9512$$

Final Settlement

The final settlement price (ticker symbol VOLS) is calculated once every trading day. The settlement value will be the Closing Volume Weighted Average Price ("Closing VWAP"), to be determined by reference to the prices and sizes of executed transactions or quotes in the thirty-two underlying NDX component options calculated at the opening of trading on the expiration date.

The Closing VWAP is calculated over a period of five minutes at the end of individual one-second time observations commencing at 9:32:00 A.M. and continuing each second for the next 300 seconds. The number of contracts traded on PHLX at each price during the observation period is multiplied by that price to yield a Reference Number. All Reference Numbers, for each second, are then summed and that sum is divided by the total number of contracts traded during the observation period to calculate a Volume Weighted Average Price (VWAP) for that observation period for that component.

Each one-second VWAP for each component option is then used to calculate the VOLQ, resulting in the calculation of 300 sequential VOLQ values. Finally, all 300 values will be arithmetically averaged (i.e., the sum of 300 Volatility Index calculations is divided by 300) and the resulting figure is rounded to the nearest 0.01 to arrive at the settlement value disseminated under the ticker symbol VOLS.

If no transaction takes place during any one-second period, the NBBO midpoint at the end of the one-second observation period will be considered the One-Second VWAP for that observation period.

Conclusion

VOLQ is calculated throughout the trading day using published, real-time bid and ask quotes on the most liquid NDX options. Since options on NDX trade on three different exchanges, these NBBO bid/ask spreads are generally tighter than the spreads of options that trade on a single exchange. As a result, VOLQ creates a clearer measure of expectations for future volatility.

Calculating implied volatility for the next 30 days requires a precisely at-the-money option expiring in exactly 30 days. However, the method to calculate VOLQ rests upon the fact that this option is not listed in the market for all prices and times. Accordingly, VOLQ requires the transformation of a series of other options that are listed on the market. The prices of these options are what is used to calculate 30-day closed-form implied volatility.

VOLQ measures what traders look at most, at-the-money volatility. By using a fixed number of options (32) the accuracy of the measurement increases; using all options listed would limit understanding of index drivers.

References

1. Brenner, M. & M.G. Subrahmanyam, 1988, "A Simple Formula to Compute the Implied Standard Deviation", Financial Analysts Journal, Sept/Oct, pp.80-83

Appendix

DERIVATION OF THE CFIV FORMULA

Starting with the standard Black Scholes formula:

$$C(S,t) = N(d_1) * S - N(d_2) * K * e^{-r(T-t)}$$

Where all variables stand for their usual meaning in the standard Black Scholes equation.

For at-the-money call options, we have:

$$S = K * e^{-r(T-t)}$$

Substituting the second equation into the first, we get:

$$C(S,t) = \left(N\left(\left(\frac{1}{2}\right) * \sigma * \sqrt{T-t} \right) - N\left(-\left(\frac{1}{2}\right) * \sigma * \sqrt{T-t} \right) \right) * S$$

Taylor's formula for small x implies:

$$N(x) = N(0) + N'(0) * x + N''(0) * \frac{x^2}{2} + O(x^3)$$

Using the Taylor series on the previous equation, we get:

$$C(S,t) = S * (N'(0) * \sigma * \sqrt{T-t} + O\left(\sigma^3 * \sqrt{(T-t)^3} \right))$$

For soon-to-expire options, we can ignore the second term and we know that for a normal distribution:

$$N'(0) = \frac{1}{\sqrt{2 * \pi}}$$

This gives us the equation:

$$C(S,t) = \left(\frac{1}{\sqrt{2 * \pi}} \right) * S * \sigma * \sqrt{T-t}$$

On rearranging, we get the equation for implied volatility:

$$\sigma = C * \frac{\sqrt{2 * \pi}}{S * \sqrt{T-t}}$$

The above equation has variables with different meaning as compared what is described in the steps of the document but it is essentially the exact equation used in calculating VOLQ.

To learn more about the VOLQ index:

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